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BOOK REVIEWS

Stability and Transition in Shear Flows. By P. J. Schmid & D. S. Henningson. Springer, 2001. 556 pp. ISBN 0-387-98985-4. £59.50 or $79.95

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While our knowledge of linear stability theory is by now fairly complete, no full understanding of the turbulent breakdown mechanism has yet been achieved. However, the past two decades have seen a number of successful attempts to bring instability theory a little closer to explaining the onset of turbulent behaviour. The book by Schmid & Henningson does precisely this: starting from classical stability theory, the authors present mostly recent developments, consider situations with higher complexity and eventually leave us at the brink of turbulence – as close as one can get at the time of writing.

Some familiarity with basic fluid dynamics and wave-like phenomena is assumed and the material is thus primarily intended for graduate students and beyond. As a fairly up to date account of issues between stability and turbulence, it will help those coming from any branch of fluid dynamics to quickly gain an overview of the latest techniques and results. Throughout, a variety of mathematical and numerical tools is introduced, each illustrated in different situations to show a range of possible applications. While some chapters serve as reference material, e.g. the final chapter on transition scenarios, it is also an excellent read from cover to cover!

The book starts with a brief introduction of governing equations and basic stability/instability, linear/nonlinear temporal/spatial concepts. Whereas the first part of the book (Chapters 2–5, about one third of the total) is focused on fundamental concepts of hydrodynamic (mostly temporal) stability theory, richly exemplified using simple parallel shear flows, the second part (Chapters 6–9) builds on these results and is devoted to more realistic, and hence more complex, situations such as three-dimensional boundary layers, spatially evolving flows, secondary intabilities and transition.

Chapter 2 is concerned with linear inviscid analysis, and illustrates the fundamental tools and results that are prerequisites for any serious shear flow analysis: modal solutions, Rayleigh's and Fjørtoft's criteria, the method of stationary phase, Laplace's transform. While linear dispersion relations and the associated eigenfunctions can be found in any standard textbook, particularly illuminating here is the explicit treatment and the corresponding illustrations of the evolution of a point-like disturbance in a boundary layer.

In Chapter 3 the stability analysis of two-dimensional parallel shear flows is carried a step further by keeping viscous terms, solving the resulting Orr–Sommerfeld and Squire equations and explicitly obtaining the spectra and eigenfunctions for Couette, pipe and Blasius boundary layer flows. In the limit of large Reynolds numbers, asymptotic techniques and critical layers are also briefly mentioned. Particularly helpful is the discussion of the sensitivity of numerically computed eigenvalues and its relationship to pseudospectra.

Chapter 4 further analyses linear viscous temporal instability with emphasis on the initial value problem rather than on the eigenvalue problem. In the first part of the book, this is certainly this reviewer's favourite chapter: topics such as non-normal operators, transient growth, optimal growth, and optimal disturbances are covered in great detail, illustrated by numerous examples drawn from the authors' own work.

The first part concludes with Chapter 5 on nonlinear stability of which, obviously, no general theory exists. Besides classical techniques such as weakly nonlinear expansions, wave interactions and bifurcation analysis, the chapter includes an enlightening discussion on nonlinear equilibrium solutions and provides a numerical strategy to compute them.

The second part of the book starts with Chapter 6 on the Falkner–Skan(–Cooke) velocity profiles: the drosophila of boundary layers with pressure gradient and crossflow. Then additional features due to body forces (related to system rotation or streamline curvature), surface tension or compressibility are analysed with the tools of the first part. In contrast, the section on unsteady flows introduces particular techniques to investigate stability of basic flows that are periodic in time or even display arbitrary time dependence.

After the so far very detailed coverage of many aspects of temporal stability, Chapter 7 now addresses growth of disturbances in space. This chapter, totalling 120 pages, is certainly the core of the book – besides being my favourite. By resorting to several model problems before addressing the full Navier–Stokes equations, the authors emphasize the particular difficulties associated with the spatial setting. Indeed, it is too often believed that temporal results can always be converted to spatial ones by a simple transformation. One may hope that the two pages on Gaster's transformation, clearly delimiting the validity of this method, will put an end to its erroneous use. Then, the evolution of perturbations in both space and time is addressed by introducing the concept of convective and absolute instabilities: features which are of primary importance to any non-Galilean-invariant open shear flow. After a review of the classical Briggs' method and the more practical cusp map procedure, these concepts are illustrated by considering the two-dimensional wake behind a cylinder and the three-dimensional boundary layer due to a rotating disk: two situations where much understanding has been gained by absolute instability analyses. The chapter continues with a treatment of the spatial initial value problem and the associated discussion of upstream and downstream responses to localized harmonic forcing. The authors then incorporate non-parallel effects, which are nearly always present in realistic shear flows, first by asymptotic multiple-scales analyses and then by parabolized stability equations, exemplified by Görtler vortices and the Blasius boundary layer (the extremely brief presentation of the obscure triple-deck theory is however not very illuminating). Then follows the spatial analogue of optimal disturbances and a brief review of global instability results, as derived from local stability analysis within the assumption of weakly diverging flows. The chapter closes with the receptivity problem, putting into practice some of the tools introduced above.

Chapter 8 brings us a step closer to transition by studying secondary instabilities that may, in turn, affect a finite-amplitude state resulting from a primary instability. The mathematical tool of choice is here Floquet theory, now usable in realistic situations due to the recently available computing power. The various types of secondary instabilities are discussed for Tollmien–Schlichting waves, streaks, Görtler and crossflow vortices, and the special case of Eckhaus instability.

The concluding Chapter 9 is based on the results and techniques collected in the previous chapters and considers how the various instability mechanisms can trigger transition to a turbulent regime. In contrast with the rest of the book, very few equations are introduced: the emphasis here is not on mathematical analysis but on describing the many possible routes that the complicated transition process may follow. This final chapter appears to be a quite comprehensive review (besides being an excellent read) of transition scenarios prevailing for two-dimensional waves, streaks, separation bubbles, Dean, Görtler and crossflow vortices etc. The chapter closes with a brief account of different models that have been used, more or less successfully, to predict transition.

On the technical side, one could criticise many overlarge or extremely small figure labels, several very badly placed equations (e.g. p. 238) and some inconsistencies in the layout. But rather than the authors, the publisher is to blame: its only contribution to this work being a photocomposed copy prepared from the authors' TEX files.

All in all, within its field, this is an extremely complete and well documented book – one which will be wanted in the library by all and on the desk (not on the shelf) by many.

Benoît Pier

The Field Theoretic Renormalization Group in Fully Developed Turbulence. By L. Ts. Adzhemyan, N. V. Antonov & A. N. Vasiliev. Gordon & Breach, 1999. 208 pp. ISBN 9056 99145 0. £62.

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Since the pioneering work of Kraichnan, Wyld, Edwards and Herring, more than four decades ago, the idea that the statistical theory of macroscopic fluid turbulence can be formulated as a version of quantum field theory has been well established. In particular, the concept of *renormalization* has taken root, just as it has in various branches of condensed matter physics where the many-body problem is the prime consideration.

However, although this analogy can be a productive one, at the same time it is essential to bear in mind the differences between turbulence (macroscopic, deterministic, classical, dissipative) on the one hand, and field theory (microscopic, random, quantum, conservative) on the other. Probably the most important of these differences is that turbulence is far from equilibrium and a turbulent fluid is characterized by a flux of energy through its internal degrees of freedom. This is the *energy cascade* which, although conservative, is an essential element in the dissipation process. In fact it is a controlling symmetry which determines the power-law behaviour of the energy spectrum (although perhaps not that of the higher moments). There is simply no analogue of this behaviour in quantum field theory; nor in its semi-classical extension to the theory of static critical phenomena. In these equilibrium problems dimensional analysis can tell one very little and power laws are controlled by the scaling dimension under renormalization group (RG) transformation.

The early work on renormalization, as applied to the turbulence closure problem, took full account of these differences. However, following the success of RG in the theory of critical phenomena, there was a growing number of papers from the late 1970s onwards, in which the authors seemed to take a cavalier attitude to such differences. This led to various writers making extravagant claims for the relevance of their work to the notorious turbulence problem. This particular activity was the subject of much controversy over the decade from the mid-1980s to the mid-1990s and, although it drew the subject of renormalization to the attention of the wider turbulence community, it undoubtedly caused harm.

The outcome seems to be that it is almost impossible nowadays to obtain a fair hearing in the turbulence community for any proposal to use RG. Indeed, it is within my experience that a paper submitted to a leading journal can be turned down unread because it contained the dreaded acronym RG in its title!

By this stage it will be clear that I approached the book that is the subject of this review with some degree of caution. However, I was pleasantly surprised to find that it was well written, clearly argued, well balanced, and showed an intelligent appreciation of fluid turbulence, as well as an obviously profound understanding of quantum field theory. Taken on its own terms, it qualifies as a good book, albeit open to some criticism. The main problem for me lies in the title: one would not describe work on (say) the Ising model as a theory of ferromagnetism. In the same way, I think that the use of the word *turbulence* in the title is unhelpful and this is only aggravated by the qualification *fully developed*.

The topic area that is the subject of this book depends on two seminal pieces of work. These are: the application of RG to stirred fluid motion by Forster, Nelson & Stephen (1976); and the field-theoretic formalism of turbulence by Martin, Siggia & Rose (1973).

The first of these extended the RG algorithm (as used in dynamical critical phenomena) to fluid motion subject to a random stirring force. We summarize this as follows. Consider the Fourier modes of the velocity field $\boldsymbol{u}(\boldsymbol{k}, t)$, defined on the wavenumber interval $0 \leqslant k \leqslant k_0$, and filtered such that $\boldsymbol{u} = \boldsymbol{u}^<$ for $0 \leqslant k \leqslant k_1$ and $\boldsymbol{u} = \boldsymbol{u}^>$ for $k_1 \leqslant k \leqslant k_0$, where $k_1 = b^{-1} \leqslant k_0$ and $b \geqslant 1$ is the *spatial rescaling factor*. The algorithm then consists of two steps:

1. Solve the Navier–Stokes equation (NSE) on $k_1 \leqslant k \leqslant k_0$. Substitute that solution for the mean effect of the high-k modes into the NSE on $0 \leqslant k \leqslant k_1$. This results in an increment to the viscosity: $\nu_0 \rightarrow \nu_1 = \nu_0 + \delta\nu_0$.
2. Rescale the basic variables so that the NSE on $0 \leqslant k \leqslant k_1$ looks like the original NSE on $0 \leqslant k \leqslant k_0$.

These steps are repeated until a fixed point is reached and this defines the renormalized viscosity.

The basic problem is the difficulty of carrying out Stage 1. The only general method available to us relies on perturbation theory; and, in the context of the NSE, that means an expansion in powers of the Reynolds number, which is normally large. This is where RG comes into its own. The *local* Reynolds number $R(k_1) = [E(k_1)]^{1/2}/\nu_0 k_1^{1/2}$, where ν_0 is the (unrenormalized) kinematic viscosity of the fluid, can be made as small as one pleases by going to either very small or very large wavenumbers. The situation is illustrated in figure 1. Forster *et al.* chose to study the low-wavenumber case (with k_0 chosen low enough to exclude the energy cascade), where one may evaluate coefficients in a perturbation series as Gaussian averages. Here the fixed point corresponds to some form of universal behaviour in the limit as $k \rightarrow 0$. In contrast, a theory of turbulence would involve elimination of bands of modes in the dissipation range, with the fixed point corresponding to the upper end of the inertial range. The basic difficulty in this case is the need for a non-trivial conditional average. Apparently the only work addressing this problem is the method of iterative averaging (see McComb & Johnston 2001 and references therein).

RG in quantum field theory is not the same as the above. It relies on the concept of renormalization invariance which in turn relies on the fact that the bare mass m_0 of a particle is not an observable. It is the renormalized mass m which is observable. In macroscopic fluid motion things are the other way round. The analogous renormalizable quantity is the kinematic viscosity ν_0 but this *is* an observable and it is the renormalized viscosity which is *not* observable. Nevertheless, the above recursive procedure permits one to derive RG equations and these, when

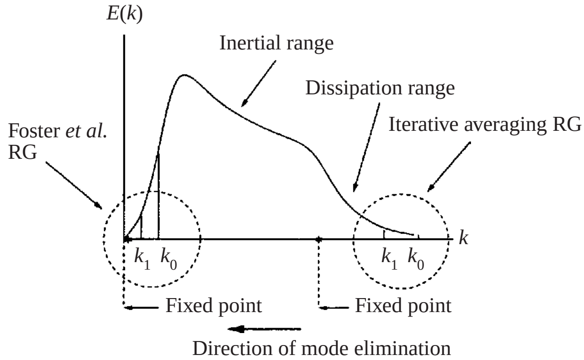

FIGURE 1. RG iterations at both low and high wavenumbers: in each case the fixed point is indicated schematically by an asterisk on the k-axis.

combined with the formalism of Martin *et al.*, allow the authors to set up a field-theoretic formalism for the stochastic NSE, and indeed to convert the work of others into the same unified format.

At 200 pages, the book is short and consists of only three chapters, preceded by a brief introduction and followed by an even briefer afterword. The introduction gives a concise but good summary of the history of RG, first in quantum field theory and then in the theory of critical phenomena. The authors point out that RG in turbulence has had a longer and less happy history (although they appear to be interpreting Kolmogorov (1941) as the beginning of this type of approach) and that most of the actual RG work has been done after the 'Golden Age' of its application to critical phenomena.

In Chapter 1, they set up their field-theoretic formalism and this is done in quite an uncompromising way. For instance, a length-scale cutoff in the noise autocorrelation is changed to a mass, as this would be the equivalent in quantum field theory. At this point (and thereafter) one wonders who do they envisage as their readership? Even the most theoretical of fluid dynamicists will find this counter-intuitive. As a result, one needs to be more than just familiar with field theory or this book is going to be very hard work.

Chapter 2 extends the analysis to composite operators, which are clearly and succinctly defined as 'any monomial or polynomial constructed from the fields or their derivatives at a single point'. In my experience, books on field theory and condensed matter theory generally leave you to work this out for yourself. This chapter claims to provide a theoretical basis for the second Kolmogorov hypothesis and also sheds light on the Yakhot–Orszag theory of the 1980s. Chapter 3 extends the analysis to applications and all the usual suspects from passive scalar convection through anisotropic turbulence to plasmas can be found here.

The formalism (for anyone with a taste for quantum field theory) is elegant, and the discussion at many points is intriguing. Perhaps this work has a contribution to make to turbulence theory but one does very much wonder about the relevance to real turbulence at this stage in its development. The key issue is that the entire theory rests on the arbitrarily chosen autocorrelation of stirring forces $d_F(k)$. If the zero-order correlation of velocities is represented by $Q_0(k) = G_0(k)d_F(k)$, where $G_0(k)$ is the viscous Green's function, this is simply not an observable and all the talk of UV and IR divergences is very misleading as, unlike in field theory, they are not inherent in the formulation but only reflect a pathological choice (albeit subject to some constraints) of $d_F(k)$. Of course renormalization leads to some $Q(k) = G(k)d_F(k)$, but everything we know about the physics of turbulence (including the earlier renormalization work) suggests that a viscous renormalization $G_0(k) \rightarrow G(k)$, cannot be the whole story. It is an article of faith in turbulence that $Q(k)$ in the inertial and viscous ranges of wavenumbers is independent of the *choice* of $d_F(k)$.

From comments made early on, and in their afterword, the authors are aware of this weakness in their position. Also, in the last paragraph of their introduction, they mention the method of *iterative averaging* and describe it as 'close in spirit to the Wilson form of the RG'. They mention it again in their afterword, but it does not appear in the text. This seems to me to be an opportunity missed. Some attempt to reconcile *iterative averaging* with the work they do discuss (which is valid only in the limit $k \rightarrow 0$: see figure 1) could have been enlightening.

To sum up, this book is well written and (with the exception of the deplorably inadequate index) well produced. It reports an activity which at a conservative estimate must have resulted in about 100 papers over two decades. Probably no other area of turbulence theory can claim such a sustained level of activity: it cannot therefore be ignored. If you are fluent in 'quantum field theory' you will enjoy it.

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